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INVESTIGATION OF TRANSONIC UNSTEADY-STATE FLOW IN THE PRESENCE OF PHASE TRANSFORMATIONS

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INTRODUCTION

Condensation of supersaturated vapor in a transonic flow can lead to an unsteady-state character of the flow. This is due to the evolution of the latent heat of condensation, to the formation of a shock wave, and to its interaction with the zone of evaporation. This phenomenon was first noted in [1, 2], in which it is shown that the character of the motion of the shock wave depends on the parameters in the initial cross section, the relative moisture content, and the contour of the nozzle. In [3] there were measured considerable pulsations of the parameters of the flow (with a frequency of 500-1000 Hz), arising with the flow of moist air and pure water vapor in air. In [4] an approximate law of similarity was introduced for the dimensionless frequency of an unsteady-state flow. In communications [5, 6] the phenomenon under consideration was studied by the method of the inversion of the action; [7, 8] give the results of theoretical calculations and an experimentally confirmed diagram, making it possible to determine the boundaries of the region of instability of the flow. It has been found recently that the frequency of the pulsations of the pressure and the density in a flow with the condensation of moist air can attain 6000 Hz. In the present work, a modification of the method of Godunov [10] is used to obtain a numerical solution of a system of equations describing an unsteady-state quasi-one-dimensional flow with spontaneous condensation in the transonic part of a Laval nozzle. Calculations of nonequilibrium unsteady-state flows in nozzles by the method of establishment have also been made previously, for example, in [11, 12] (mixed flow in nozzles), [13] (flow taking account of vibrational relaxation and nonequilibrium chemical reactions), and [14] (two-phase flow in a nozzle, with disagreement of the phases with respect to velocities and temperatures). The specific characteristic of the present problem consists in the fact that, during the process of establishment with steady-state initial and boundary conditions, the limiting state is not steady-state; however, a known periodicity is observed.

1. Let us consider the unsteady-state quasi-one-dimensional flow of supersaturated vapor in a Laval nozzle, without taking account of viscosity, thermal conductivity, or radiation. We assume that the velocities of the phases are identical, and that the condensation is spontaneous. The dependence of the area of the transverse cross section of the nozzle on the coordinate x, varying along the axis, is given by the function F(x); here x = 0 corresponds to the minimal cross section of the nozzle. Let p be the pressure, ρ the density of the mixture, u the velocity, and t the time; the parameters of the condensing phase have the superscript zero. The basic equations of the conservation of mass, momentum, and energy can be written in the form

$$\frac{\partial}{\partial t}(\rho F)+\frac{\partial}{\partial x}(\rho u F)=0;$$

$$\frac{\partial}{\partial t}(\rho u F)+\frac{\partial}{\partial x}[(p+\rho u^2)F]=p\frac{\partial F}{\partial x};$$

$$\frac{\partial}{\partial t}\left[\rho F\left(h-\frac{p}{\rho}+\frac{u^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho u F\left[h+\frac{u^2}{2}\right)\right]=0. \quad (1.1)$$

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The enthalpy of the mixture h depends on the mass concentration of the vapor β and is determined by the formula

$$h = \frac{\varkappa}{\varkappa - 1} RT + (1 - \beta) c^0 T^0, \quad RT = \frac{p}{\rho}, \tag{1.2}$$

where $\varkappa$ is the adiabatic index of the vapor; T is the temperature; c^0 is the specific heat capacity of the liquid phase; R is the gas constant.

We assume that the forming particles have a spherical form and that the rate of their growth $\dot{r} = dr/dt$ does not depend on the radius r. In this case, the spontaneous condensation of the flowing vapor is determined by the system of equations of [15, 16], for the purposes of the present work written in the form

$$\frac{\partial}{\partial t}(\rho F \Omega_k) + \frac{\partial}{\partial x}(\rho u F \Omega_k) = \rho F \omega_k \quad (k = 0, 1, 2, 3); \tag{1.3}$$

$$\omega_k = \frac{I}{\rho} r_*^k + k\dot{r}\Omega_k; \quad \beta = -\frac{4\pi}{3} \rho^0 \Omega_3,$$

where r_* is the radius of a nucleus; Ω_k are auxiliary functions, introduced in [15]; I is the rate of formation of the nuclei, which, in the present work, is determined by the Frenkel'–Zel'dovich formula [17].

For the rate of growth of a drop the Knudsen formula can be used,

$$\dot{r} = \frac{\alpha}{\rho^0 (2\pi RT)^{1/2}} \left[p - \left(\frac{T}{T^0}\right)^{1/2} p_s(T^0)\right], \tag{1.4}$$

and, for determination of the temperature of the drop, the equation [16, 18]

$$\alpha\left(\frac{T^0}{T} - \frac{2L}{RT}\frac{\varkappa - 1}{\varkappa + 1}\right)\left[1 - \left(\frac{T}{T^0}\right)^{1/2} \frac{p_s(T^0)}{p}\right] - \alpha\left[1 - \left(\frac{T^0}{T}\right)^{1/2} \frac{p_s(T^0)}{p}\right] + (1 - \alpha)\gamma\left(\frac{T^0}{T} - 1\right) = 0, \tag{1.5}$$

where p_s is the pressure of the saturated vapors above the flat surface of the duct; L is the heat of condensation; α and γ are the coefficients of condensation and thermal accomodation.

As has been shown by a comparison of the results of theoretical calculations with experimental data [19], good agreement between the location of the region of a jump in the condensation and the distribution of the static pressure along the flow lines is observed with the values $\alpha = 0.04$ and $\gamma = 1$. If into the power exponent of the Frenkel'–Zel'dovich formula [17] for the rate of formation of the nuclei we introduce the semiempirical coefficient n, good agreement between theory and experiment can also be attained with respect to the dispersivity [6].

In the calculations it was assumed that $\alpha = 0.04$, $\gamma = 1$, and $n = 1.8$.

The initial conditions were the following: with $t = 0$ all the parameters were constant and equal to their values in the initial cross section of the nozzle $x = x_0$ (with $u = 0$); the pressure in the outlet cross section of the nozzle $x = x_N$ was equal to the pressure of the surrounding medium p_a.

2. The system of equations (1.1)-(1.5) was solved in an electronic computer using a method [10] based on the principle of establishment, without the evolution of shock waves. The whole nozzle was divided into N sections. To the parameters at the mesh points there were assigned the indices 0, 1, 2, ..., m, ... N, and, at the middles of the segments between mesh points with the coordinates x_m and x_{m+1}, the fractional index $m + 1/2$. For quantities corresponding to the moment of time t, this index was shifted downward, and, for the moment of time $t + \Delta t$, upward. The calculating scheme, by which the values at the moment of time t and the parameters at the boundary are used to find the values at the succeeding moment of time $t + \Delta t$, was obtained by the integration of the left- and right-hand parts of the differential equations (1.1), (1.3) along the contour of an elementary cell, followed by application of the theorem of the mean. The following system of difference equations was obtained for determination of ρ, u, h, and Ω_k:

$$\rho^{m+\frac{1}{2}} = \rho_{m+\frac{1}{2}} - \frac{\Delta t}{\Delta x_m F_{m,m+1}} [(\rho u F)_{m+1} - (\rho u F)_m]; \tag{2.1}$$

$$(\rho u)^{m+\frac{1}{2}} = (\rho u)_{m+\frac{1}{2}} - \frac{\Delta t}{\Delta x_m F_{m,m+1}} [(p + \rho u^2)_{m+1} F_{m+1} - (p + \rho u^2)_m F_m] +$$

$$+ \frac{\Delta t}{\Delta x_m F_{m,m+1}} \left(p^{m+\frac{1}{2}} + p_{m+\frac{1}{2}}\right); \quad \left[\rho\left(h - \frac{p}{\rho} + \frac{u^2}{2}\right)\right]^{m+\frac{1}{2}} =$$

$$= \left[\rho\left(h - \frac{p}{\rho} + \frac{u^2}{2}\right)\right]_{m+\frac{1}{2}} - \frac{\Delta t}{\Delta x_m F_{m,m+1}}\left\{\left[\rho u F\left(h + \frac{u^2}{2}\right)\right]_{m+1} -\right.$$

$$\left.- \left[\rho u F\left(h + \frac{u^2}{2}\right)\right]_m\right\}; \ (\rho\Omega_k)^{m+\frac{1}{2}} = (\rho\Omega_k)_{m+\frac{1}{2}} - \frac{\Delta t}{\Delta x_m F_{m,m+1}}[(\rho u F\Omega_k)_{m+} -$$

$$- (\rho u F \Omega_k)_m] + \frac{\Delta t}{2}\left[(\rho\omega_k)^{m+\frac{1}{2}} + (\rho\omega_k)_{m+\frac{1}{2}}\right] \quad (k = 0, 1, 2, 3).$$

Here $\Delta x_m = x_{m+1} - x_m$, $2F_{m,\,m+1} = F_m + F_{m+1}$. The remaining parameters are found from (1.2)-(1.5).

Formulas (2.1) are used to determine the parameters with fractional indices. Values with whole indices are found using formulas for the decomposition of an arbitrary discontinuity [10], modified for application to the nonequilibrium flow under consideration. The modification involved only methods for determining the parameters Ω_k, characterizing the relaxation process, and the parameters at the boundary (with $x = x_0$ and $x = x_N$). Since the time required for the establishment of equilibrium with respect to translational and rotational degrees of freedom, characterizing the thickness of the gas dynamic discontinuities, is considerably less than the time of the relaxation process of mass transfer, we shall assume that Ω_k does not change with a transition through the gasdynamic discontinuities. Thus, the values of the parameters Ω_k with $x = x_m$ will depend only on the sign of the velocity of the contact discontinuity U_m at this point. The conditions of a decomposition of the discontinuity for Ω_k can be written in the form

$$(\Omega_k)_m = (\Omega_k)_{m-\frac{1}{2}} \quad \text{with} \quad U_m \geqslant 0;$$

$$(\Omega_k)_m = (\Omega_k)_{m+\frac{1}{2}} \quad \text{with} \quad U_m < 0.$$

To determine the parameters at the boundaries ($x = x_0$ and $x = x_N$) several additional cells are connected to the nozzle on the left and the right, and we assume that the flow in these sections is isentropic. By virtue of the statement of the problem, in the initial cross section of the nozzle, there is the known right-hand Riemann invariant and the value of $S_0 = p_0\rho_0^{-\varkappa}$, and the left-hand invariant is calculated from the parameters of the adjacent cell at the moment of time t. This permits calculating all the parameters in the inlet cross section [11, 12]. An analogous method is used to determine the parameters in the outlet cross section of the nozzle, with the difference that the roles of the left- and right-hand Riemann invariants change places.

The nozzle was divided into 40-80 segments of different length. The minimal spacing Δx was so selected that large gradients of the parameters p, u, ρ, Ω_k were to be expected (in the region of the minimal cross section of the nozzle or in the presumed region of the appearance of gasdynamic discontinuities). The Courant – Friedrichs – Levy criterion was used for the stability of the solution [20].

3. The problem was solved in an M-222 digital computer. The pressure and temperature in the initial cross section of the nozzle $x_0/l_* = -3.8$ (l_* is the breadth of the minimal cross section) were the following: $p_0 = 1$ atm (abs.), $T_0 = 373$°K. An investigation was made of the expansion of supersaturated water vapor in three nozzles. The results for all the cases are shown in Fig. 1a-c.

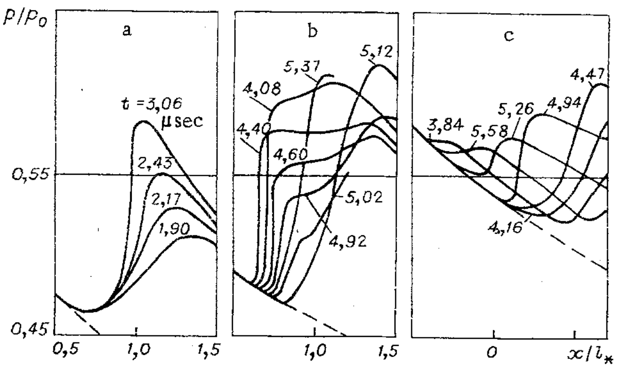

Fig. 1

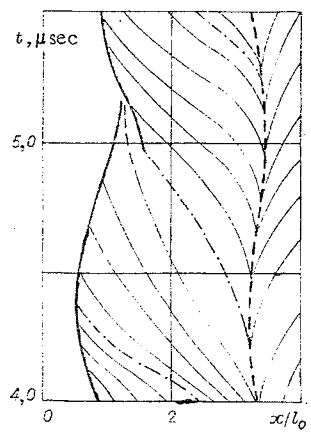

Fig. 2

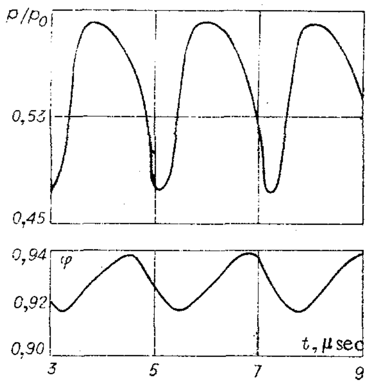

Fig. 3

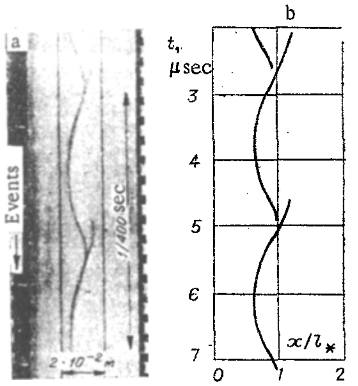

Fig. 4

Figure 1a shows curves of the distribution of the pressure along the axis of the first nozzle at different moments of time (the dashed curve corresponds to a flow without condensation). It can be seen that the process is fully established and that, with $t \approx 3.06$ μsec, there is formed a stable shock wave, due to the evolution of heat with condensation. Downstream from the shock wave there is a region of subsonic flow, in which the supercooling is somewhat less than ahead of the shock wave; however, it ensures the further growth of the forming nuclei. Such flow conditions with a stationary shock wave have been observed experimentally [6, 7].

Figure 1b shows the distribution of the pressure at different moments of time for the second nozzle with a lesser degree of expansion with the same conditions in the initial cross section. It can be seen that the process is not fully established: the intensity of the shock wave arising at first rises, as a result of which it is displaced countercurrent, and then decreases, which brings about its displacement in the reverse direction and complete damping. After this, a new shock wave is formed and the process is repeated cyclically.

In the cases under consideration, the shock wave did not reach the minimal cross section of the nozzle; therefore, a calculation was made for a nozzle with a still smaller degree of expansion, which permitted extending the zone of condensation right up to the minimal cross section. Figure 1c shows the distribution of the pressure for this case; the shock wave is displaced upstream from the minimal cross section of the nozzle and is damped.

Let us consider the field of the flow immediately in the zone of condensation behind the shock wave (Fig. 2). The heavy line denotes the front of the shock wave, and the thin solid lines are the characteristics of the second family. Behind the shock wave, the flow is subsonic; however, as a result of the supply of heat due to condensation it is accelerated to a sonic velocity, and then, as a result of an increase in the degree of expansion of the nozzle, becomes supersonic. Thus, in addition to the usual sonic line in the critical cross section of the nozzle, downstream there may exist a second sonic line (see Fig. 2, dashed line).

The whole flow in the region of condensation can be divided into alternating compression and rarefaction waves. In Fig. 2, these regions are separated by the characteristics of the second family, denoted by a dashed-dot line. The interaction between the compression wave and the shock wave leads to an increase in the intensity of the latter and to its countercurrent displacement. This partially eliminates the intersection in the condensation zone, the supply of heat becomes less intense, and the region generating a compression wave vanishes. After this, the shock wave starts to interact with the rarefaction wave. The rate of displacement of the shock wave with respect to the nozzle decreases and becomes equal to zero. At this moment, its intensity can be determined using known relationships for the calculation of steady-state direct shock waves, with a given spacing of the Mach number of the oncoming flow. If the shock wave has attained the minimal cross section of the nozzle, it starts to move downstream and is damped. If it has passed through the minimal cross section, damping takes place with a simultaneous displacement upstream. A decrease in the intensity leads to an increase in the supersaturation in the condensation zone, the supply of heat becomes more intense, and a compression wave is again formed, then going over into an alternating shock wave. The process repeats itself cyclically.

It is of interest to note that the processes of the damping of the preceding shock wave and the formation of the succeeding one overlap in time. The succeeding shock wave originates downstream from the preceding. As a result, in the region of the flow between them, a third periodically appearing and vanishing sonic line is formed (see Fig. 2, dashed lines).

If we consider a fixed cross section of the nozzle, then the parameters of the flow in it will vary with a definite frequency. For example, the pulsations of the static pressure near the minimal cross section of the nozzle attain 35-40%, and the pulsations of the Mach number about 20%. The whole cycle repeats itself in approximately 2 μsec, i.e., the frequency of the pulsations attains 500 Hz. The upper curve of Fig. 3 shows the dependence of the pressure on the time for the second nozzle in the cross section $x/l_* = 0.835$.

An unsteady-state character of the flow in the condensation zone can lead to pulsations of all the flow parameters, including the mass flow rate and the specific momentum. Since in the first two nozzles the shock wave does not reach the critical cross section, the mass flow rate of the gas remains constant. However, in the third nozzle investigated the shock wave is shifted into the subsonic part, as a result of which there are pulsations of the mass flow rate. Just such an unsteady-state character was noted in [3, 4]. Figure 3 gives a curve of the change in the draft coefficient in the outlet cross section of the third nozzle. The calculated Mach number at the outlet for a flow of water vapor without condensation was equal to 1.3.

In the experimental study of unsteady-state flows it is usual to use Töpler moving-picture photos, obtained with operation of the moving-picture camera under recording conditions. As a result, the photos are extended in time. Figure 4a gives a Töpler moving-picture photo for the second nozzle. The same figure shows the corresponding curve (Fig. 4b), obtained by calculation. In spite of a certain difference in the frequency, the qualitative agreement between the experiments and the results of the calculations is obvious.

Thus, with spontaneous condensation in the transonic part of a Laval nozzle, the following flow pictures may be observed: a) a stationary shock wave is formed, the flow is fully established, and the mass flow rate is determined from the condition of isentropic expansion; b) in the supersonic part of the nozzle, shock waves are formed periodically and the flow is unsteady-state; however, the mass flow rate remains unchanged; c) the periodically forming shock waves are shifted into the subsonic part of the nozzle, the flow is unsteady-state and is accompanied by periodic pulsations of all the gasdynamic parameters, including the mass flow rate.

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STATISTICAL CHARACTERISTICS OF THE DIFFUSION OF A CHEMICALLY ACTIVE ADDITIVE IN A TURBULENT MIXING ZONE

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In the article a numerical solution of the connected system of the equations of turbulent transfer for the fields of the velocity and concentration of a chemically active additive is used to calculate a number of the second moments of the concentration field in a flat mixing zone. The system of transfer equations is derived from the equations for a common function of the distribution of the fields of the pulsations of the velocity and the concentration [1] and is simplified in the approximation of the boundary layer. A closed form of the transfer equations is obtained on the level of three moments, using the hypothesis of four moments [2] and its generalized form for mixed moments of the field of the velocity and the field of a passive scalar. The differential operator of the closed system of the equations of turbulent transfer for the fields of the velocity and the concentration is found by a method of closure not of the parabolic type but of a weakly hyperbolic type [3]. An implicit difference scheme proposed in [4] is used for the numerical solution. The results of the numerical solution are compared with the experimental data of [5].

1. System of Equations for the Moments of the Field of the Concentration

The turbulent diffusion of a dynamically passive additive in a free inhomogeneous turbulent flow of an incompressible liquid is considered in an Euler description. The dynamic passivity of the additive postulates that the field of the velocity $\vec{u}(\vec{x})$ does not undergo any appreciable effect from the side of the process of turbulent diffusion of the additive. The additive can react chemically with the medium of the flow. The chemical reaction under consideration can be assumed to be passive, which can be regarded as justified for "weak" chemical reactions in the flow (taking place relatively slowly and quietly) and small concentrations of the impurity. It is assumed that the fields of the pulsations of the velocity and concentration of the additive can be described by a common distribution function, satisfying some "kinetic" equation [1]. The equations for the moments of the field of the concentration in a free inhomogeneous turbulent flow are de-

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